EE219B: Homework 7

Professor Robert K. Brayton

Due March 14 (Thursday)*

1. Let $f : B^m \to B$. Define $S_x f = f_x + f_{\bar{x}}$, and $C_x f = f_x f_{\bar{x}}$. Let $\vec{x} = \{x_1, \cdots, x_k\}$. Define $S_{\vec{x}} f = S_{x_1} S_{x_2} \ldots S_{x_k} f$. Similarly define $C_{\vec{x}} f = C_{x_1} C_{x_2} \ldots C_{x_k} f$.

   (a) Prove that the operators S_x and C_x are duals in the following sense: $C_x f = \overline{S_x \bar{f}}$

   (b) Prove that $S_x f$ is the smallest function independent of x which contains f. Similarly, prove that $C_x f$ is the largest function independent of x which is contained in f.

2. Let $f : B^m \to B$ be a logic function with don't care $d : B^m \to B$. Let $\vec{x} = \{x_1, \ldots, x_m\}$ be the set of variables in the expression for f. $\vec{x}$ is called the *support* of f. $g : B^m \to B$ is said to be *compatible* with (f, d) if $f \subseteq g \subseteq f + d$. Prove the following statements:

   (a) There exists a function compatible with (f, d) with support $\vec{x}_0 \subseteq \vec{x}$ if and only if $S_{\vec{x}'} f \subseteq f + d$, where $\vec{x}' = \vec{x} - \vec{x}_0$.

   (b) There exists a function compatible with (f, d) with support $\vec{x}_0 \subseteq \vec{x}$ if and only if $f \subseteq C_{\vec{x}'}(f + d)$, where $\vec{x}' = \vec{x} - \vec{x}_0$.

   (c) If f^* with support $\vec{x}_0 \subseteq \vec{x}$ is compatible with (f, d), it is also compatible with $(S_{\vec{x}'} f, C_{\vec{x}'}(f + d))$.

   What can you conclude about the maximum don't care set for any function f^* with support $\vec{x}_0 \subseteq \mathbf{x}$ compatible with (f, d)?

3. Let $x = abeg' + dfg + abe'g + ade' + aceg' + ace'g + afg' + de'g + acfg + abfg + deg'$.

   (a) Do $GFACTOR(x)$, where $CHOOSE\_DIVISOR$ is $CHOOSE\_LITERAL$ and $DIVIDE$ is the algebraic divisor. Show the results at each step.

   (b) Repeat (a) by taking a level 0 kernel as a divisor. Show the results at each step.

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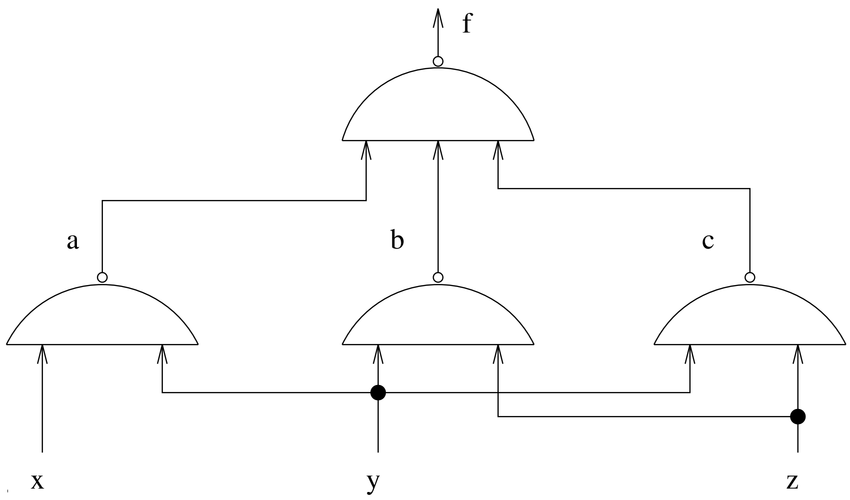

Figure 1: `NAND` based network for Problem 4

4. (a) For the circuit given in Figure 1, compute the MSPF for each node.
   (b) Using transduction, compute the CSPF for nodes a, b, c. Transduction was formulated for `NOR` gates in class, you will have to extend the results to `NAND` gates. (Use the order $a \prec b \prec c$).

5. Given a cut in a boolean network as shown in Figure 2 denote by $[m_0], ..., [m_q]$ the characteristic functions representing the equivalence classes of the cut. These characteristic functions represent set of minterms in the space B^p that produce the same outputs. Express the observability don't care set at node y_1 in terms of these equivalence classes and prove that your representation is correct.

6. Consider the following network whose primary inputs and primary output are $\{x_1, x_2, x_3\}$ and z respectively.

$$
\begin{aligned}
z &= y_1 + y_2 + y_3 \\
y_1 &= x_1 x_2 \\
y_2 &= x_2 x_3 \\
y_3 &= x_1 x_3
\end{aligned}
$$

   Assume that there is no external don't care for z. Compute the CSPFs of y_1, y_2 and y_3 using the CSPF algorithm for general networks. Use the ordering $y_1 < y_2 < y_3$.

7. Consider a network shown in Figure 3. The goal of this problem is to compute the compatible set of permissible functions for y_3, using the BDD-based technique for computing CSPFs.

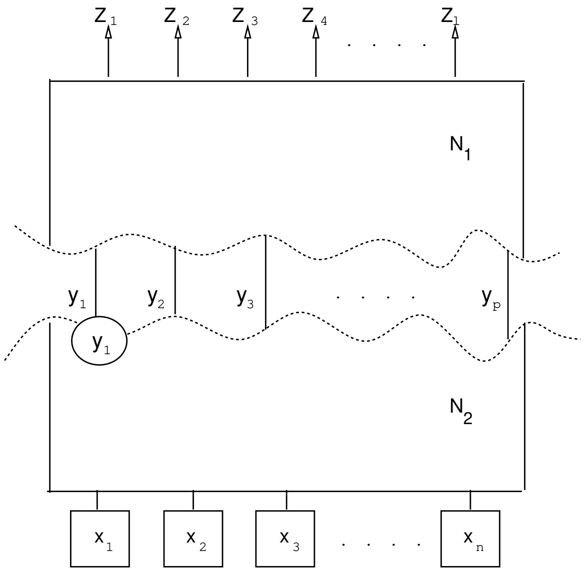

Figure 2: Equivalence Classes and ODC's of a Cut

Given:

$$
\begin{aligned}
T &= \{y_4, y_5, y_6\} \\
V &= \{y_1\} \\
y_j &= y_3 \\
f_4^C &= y_1 + y_3 \\
f_5^C &= \overline{y_3} \\
f_6^C &= y_4 y_5 \\
\mathbf{y} &= \langle y_2, x_3 \rangle \\
\mathbf{x} &= \langle x_1, x_2, x_3 \rangle \\
\mathbf{z} &= \langle z \rangle \\
\mathbf{v} &= \langle y_1 \rangle
\end{aligned}
$$

(a) Compute $g_V(\mathbf{x})$, $g_Z(\mathbf{x})$, $g_{fi}(\mathbf{x})$.

(b) Compute $(\mathbf{y} \equiv g_{fi}(\mathbf{x}))$. Note that $\equiv$ translates to an XNOR. Since $\mathbf{y}$ has two components, you do the XNOR separately for each component, and then AND the results.

(c) Compute $Z^C((\mathbf{v}, y_j), \mathbf{z})$.

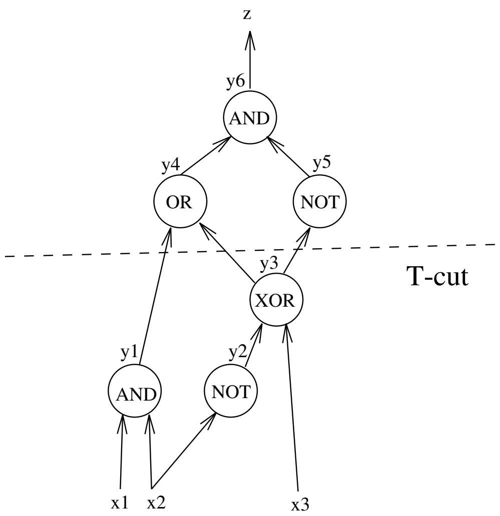

Figure 3: Circuit for T-cut problem

(d) Compute $Z^C((g_V(\mathbf{x}), y_j), \mathbf{z})$.

(e) Compute $(\mathbf{z} \equiv g_Z(\mathbf{x}))$.

(f) Compute $F_3^C((y_2, x_3), y_3) = \forall x_1, x_2, z[((\mathbf{y} \equiv g_{fi}(\mathbf{x})) \wedge (Z^C((g_V(\mathbf{x}), y_j), \mathbf{z}))) \Rightarrow (\mathbf{z} \equiv g_Z(\mathbf{x}))]$

(g) By interpreting $F_3^C((y_2, x_3), y_3)$, describe which functions can be used at y_3.