EE290LS - Midterm 1

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NAME:

problem	maximum points	your points
problem 1	15	
problem 2	15	
problem 3	20	
problem 4	25	
problem 5	25	
total	100	

1. (a) Let $f = abd + \overline{c}d$. Draw the ROBDD for f using variable ordering

    i. $a < b < c < d$

    ii. $d < c < b < a$

    Use complement edges.

   (b) The odd parity function f on n variables is a function $f : B^n \to B$ such that for any $x \in B^n$, $f(x) = 1$ if and only if the number of the variables set to one in x is odd. How many non-terminal nodes are required to represent this function by ROBDD using complement edges? Note that the size of the ROBDD is independent of variable ordering since the function is symmetric. Draw the ROBDD of the function for $n = 2$ using complement edges.

2. Suppose a cube $c = x\overline{y}zw$ is expanded against an offset cover. The following is the blocking matrix we got.

x	y	z	w
1	0	0	0
0	1	0	0
0	0	1	1

   (a) What vertices in this Boolean space B^4 are guaranteed to be in the offset? (Hint: Use information extracted from the blocking matrix.)

   (b) List all the maximally expanded cubes of c.

3. Let X_1, X_2 and X_3 be multi-valued variables. The domains of X_1, X_2 and X_3 are $\{0, 1, 2, 3\}$, $\{0, 1, 2\}$ and $\{0, 1\}$ respectively. The following is a cover $\mathcal{F}$ of a function f in the positional notation.

X_1				X_2			X_3	
1	1	1	1	0	0	1	1	1
1	1	1	1	1	0	0	0	1
0	1	0	0	0	1	0	0	1
1	1	0	0	1	0	0	1	0

   (a) Is $\mathcal{F}$ a weakly unate cover in X_1 ?

   (b) Is f a weakly unate function in X_1 ?

   (c) Is $\mathcal{F}$ a strongly unate cover in X_1 ?

(d) Is f a strongly unate function in X_1 ?

(e) Is $\mathcal{F}$ a strongly unate cover in X_2 ?

(f) Is f a strongly unate function in X_2 ?

Do not simply answer yes or no.

4. The figure in the next page shows an undirected graph $G = (V, E)$, where V and E are the set of all the vertices and the set of all the edges in G, and a set of undirected graphs $P = \{P_1, P_2, P_3, P_4\}$, where $P_i = (V_i, E_i)$ for $i = 1, \ldots, 4$. Each node in G and $P_i (i = 1, \ldots, 4)$ is colored either in white or in black as shown in the figure. A tuple (P_i, V'), where $P_i \in P$ and $V' \subseteq V$, is called a match of the graph G if and only if $|V_i| = |V'|$ and there exists a one-to-one function $f : V_i \rightarrow V'$ such that

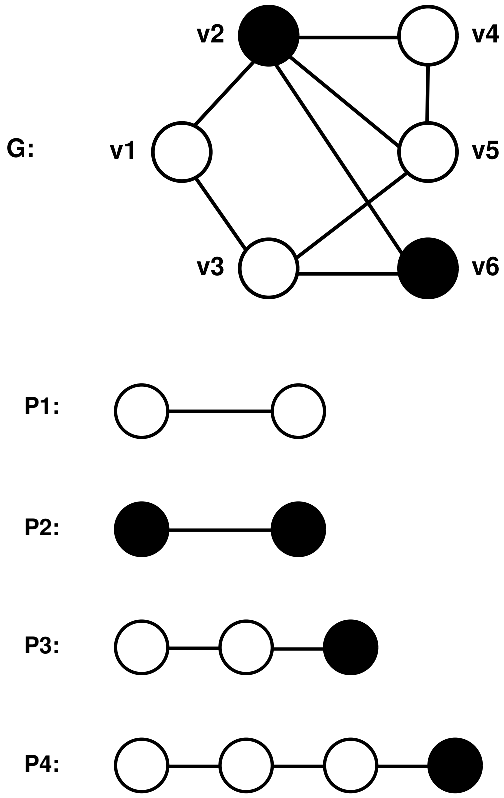
   (a) Each vertex $u \in V_i$ is colored in the same color as $f(u) \in V'$, and

   (b) If there is an edge $(u, v) \in E_i$, then $(f(u), f(v)) \in E$.

   If (P_i, V') is a match of the graph G, then vertices V' are said to be covered by the match. A set of matches is called a cover of the graph G if and only if every vertex of G is covered by a match in the set. Note that a vertex can be covered more than once.

   For example, $(P_1, \{v_1, v_3\})$ is a match of G and $\{(P_1, \{v_1, v_3\}), (P_2, \{v_2, v_6\})$ $(P_3, \{v_4, v_5, v_2\})\}$ is a cover of G.

   The *minimum graph covering problem* of G is to find a cover of G which contains the minimum number of matches. Formulate this problem as a unate covering problem and solve it. Show the cyclic core of the covering table.



5. Consider the *sum-to-one subsets* problem defined as follows.

   Definition 1 *Given a set of completely specified functions $F = \{f_1, f_2, \ldots, f_k\}$ where $\sum_{i=1}^{k} f_i = 1$ and each $f_i (i = 1, \ldots, k)$ is a function of $X = \{x_1, x_2, \ldots, x_n\}$, the sum-to-one subsets problem is to find all the minimal subsets of the functions in F which add up to 1 (i.e. tautology).*

   We want to design an efficient ROBDD-based algorithm for this problem.

   (a) How is IRREDUNDANT of Espresso related with the sum-to-one subsets problem? Can you perform IRREDUNDANT given a procedure to solve the sum-to-one subsets problem exactly?

   (b) For each function f_i, we introduce a Boolean variable y_i to represent whether the function is selected as a member of a subset, i.e. $y_i = 1$ if and only if the subset contains f_i. Prove the following lemma, which is essential in designing our procedure.

Lemma 1 *Let* $F = \{f_1, f_2, \ldots, f_k\}$ *be a set of functions of variables* $X = \{x_1, x_2, \ldots, x_n\}$*. Let* $Y = \{y_1, y_2, \ldots, y_k\}$ *be Boolean variables where* $X \cap Y = \emptyset$ *and* $K = \{1, \ldots, k\}$*. Then the following conditions are equivalent for* $\forall \hat{K} \subseteq K$.

i. $\sum_{i \in \hat{K}} f_i = 1$

ii. $\prod_{i \in \hat{K}} y_i \Rightarrow \sum_{i \in K} y_i f_i$

where $\sum$ *and* $\prod$ *are Boolean OR and Boolean AND respectively.*

(c) Show that the sum-to-one subsets problem can be solved by the prime generation of a function. (Hint: Use the previous lemma.)

(d) Show that all the primes of our interest can be generated easily once the ROBDD of the function in (c) is constructed. (Hint: Use a variable ordering where all the y-variables precede x-variables.)