EE219B: Homework 3

Professor Robert K. Brayton

Due in class on February 13, Tuesday. *

1. Consider a cover $\mathcal{F} = \{a'bc'd, a'b'c', a'cd, abd, bcd\}$ of a completely specified function f, whose offset $\mathcal{R}$ is $\{ab', bd', cd'\}$.

   (a) Form the blocking matrix B of cube $a'bc'd$. Derive $\overline{g}(y)$ from B.

   (b) Write down the covering function $h(y)$ for cube $a'bc'd$.

   (c) Expand $a'bc'd$ following the algorithm given in class. Explain why this particular expansion is a reasonable choice.

2. (a) Prove the following, where F is a cover:

   $$\text{SCCC}(F) = \text{SCC}(x\,\text{SCCC}(F_x) + \overline{x}\,\text{SCCC}(F_{\overline{x}}))$$

   This is the main fact used to prove that the merging step of the SCCC algorithm produces the desired result.

   (b) Let $\mathcal{F} = \{ab, ac, a'b', a'c, d'\}$ be a cover of a completely specified function. Reduce cube d' using the SCCC algorithm.

3. Consider two completely specified functions $f_1 = a'b'c' + a'bc' + ab'c' + abc$ and $f_2 = a'b'c' + a'bc' + ab'c' + abc' + abc$.

   (a) Generate all the primes of

   i. f_1,
   ii. f_2,
   iii. the two-output function $\{f_1, f_2\}$.

   (b) Construct an exact minimum cover of

   i. f_1,
   ii. f_2,
   iii. the two-output function $\{f_1, f_2\}$.

   How do you interpret these results? The first two give you one implementation of f_1 and f_2 while the last one gives another. Do you get a better result with multiple output minimization than with separate minimization of f_1 and f_2? If so, why?

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4. Let f be a function with multi-valued input $f : P_1 \times P_2 .... \times P_n \rightarrow B$. Prove that if f is strongly unate then every prime of f is essential.

5. For multiple valued functions, we have defined the cofactor of a cover with respect to a cube d, operationally in terms of cofactoring the cubes of the cover by the cube. (See page 13 of the second multi-valued logic handout.) Prove that if we have two covers representing the same completely specified mv-function, $F1$ and $F2$ then $F1_d$ represents the same function as $F2_d$, i.e. if $F1 = f$ and $F2 = f$ then $F1_d = f_d$ and $F2_d = f_d$, so that f_d is well defined.