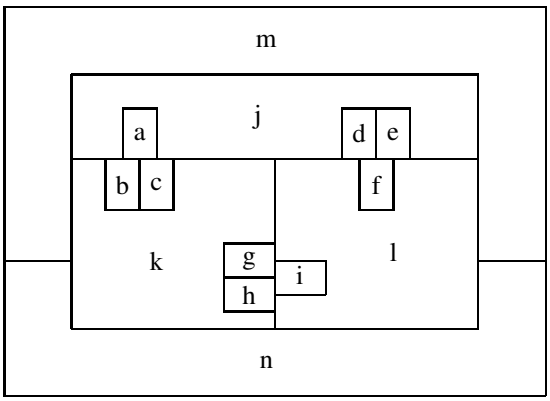
EE219B: Homework 2

Professor Robert K. Brayton

Due in class on February 6 (Tuesday)*

1. Given a map, consider the problem of finding the minimum number of colors necessary to make an assignment of colors to the regions on the map so that every region has one color and no two adjacent regions have the same color. Formulate the minimum map coloring problem as a logic minimization problem. Apply your formulation to the following map and use ESPRESSO[1] to find a minimum coloring for the map. Use ESPRESSO's normal default option in one run and the *exact* option in another run. How do you interpret the answer given by ESPRESSO? What happens with this interpretation when a region is assigned more than one color?
   (HINT: Try to formulate the problem by specifying it as a minimization of an incompletely specified function with the offset giving the conditions that no two adjacent regions have the same color, and the onset giving the conditions that every region must have a color.)



2. In this problem we apply the unate recursive paradigm to complementation.

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[1]ESPRESSO is available as `/usr/eesww/bin/espresso` on cadgroup workstations. You can find the manual of ESPRESSO in `/projects/sis/sis/common/man/man1/espresso.1`.

(a) Prove that $\overline{f} = x\overline{(f_x)} + \overline{x}\overline{(f_{\overline{x}})}$. This result directly yields a merging process for complementation using the unate recursive paradigm.

(b) Use the unate recursive paradigm to complement $f = xy'z' + wxy' + wy'z + wx'z + wyz + xyz + w'xy'z$ using the merging process inferred from (a). Show results at each step. (i.e. "personality matrix" B[2], all minimal column covers, etc.) Also, complement f by applying DeMorgan's Law.

3. Let $f = w'x'y'z' + wx'z' + wx + w'x'yz$ with the don't-care set $d = w'xyz' + wx'yz + w'xyz$. Use the Quine-McCluskey procedure to minimize f. Show results at each step. (i.e. list the primes, give the matrix, and show how the minimum cover is found.)

4. (Theorem 5 on page 32 of the second handout) Prove: p is a prime of f iff it is maximal among the set consisting of:

   (a) $p = qx$, q is a prime of f_x, $q \not\subseteq f_{\bar{x}}$

   (b) $p = r\bar{x}$, r is a prime of $f_{\bar{x}}$, $r \not\subseteq f_x$

   (c) $p = qr$, q is a prime of f_x, r is a prime of $f_{\bar{x}}$

5. Prove that the Shannon cofactor f_x is a cover of the incompletely specified function $(fx, \bar{x}, \bar{f}x)$ given by the generalized cofactor $co(f, x)$, i.e.

$$fx \subseteq f_x \subseteq fx + \bar{x}.$$

   Prove that f_x is the unique cover of $co(f, x)$ independent of x.

6. Consider an incompletely specified function where the don't care set is given by the cubes $abcd$ and $b'c'd$, and the following is a cover of the onset, and possibly some of the don't care set:

	a	b	c	d
1	2	0	0	2
2	2	0	2	1
3	1	0	2	2
4	1	2	2	0
5	2	1	2	0
6	2	2	0	0
7	1	2	1	2

   The variables of the function are a, b, c, d, and the above cover has 7 primes, numbered as shown. We want to compute an irredundant cover for the incompletely specified function, starting from the above cover. Let y_i be a new variable for cube c_i (as in class).

[2]The matrix B on page 18 of the second handout is called a "personality matrix".

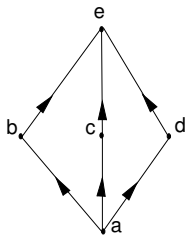

Figure 1: Non-distributive Lattice

(a) Write down the function $g(y)$, by inspecting the above cover.

(b) Determine the $\overline{g_1}, \overline{g_2}, \overline{g_3}$, and $\overline{g_5}$ components of the function $\overline{g}(y)$ using the modified tautology algorithm shown in class. Use the unate reduction technique during the recursion, where possible.

(c) From the part of $\overline{g}(y)$ computed above, write down the personality matrix B. Note that the personality matrix is just the blocking matrix of the minterm y where each y_i is 1, relative to the cubes of $\overline{g}(y)$. The minimum column cover of B gives the minimum cover of the incompletely specified function, *starting from* the given cover (you don't need to find the minimum column cover).

7. This problem concerns Boolean algebra:

(a) Using the postulates of a Boolean algebra (see Boolean algebra notes for the postulates) show that for every element $b \in \mathbf{B}$, its complement, b', is uniquely defined.

(b) Given a Boolean algebra $(\mathbf{B}, +, \cdot, 0, 1)$, let $a, b \in \mathbf{B}$. Prove that $(a \wedge b)' = a' \vee b'$.

(c) Shown in Figure 1 is the Hasse diagram for a lattice (for any two elements x and y, $x \leq y$ iff there is a directed path from x to y in the Hasse diagram). Show that the given lattice is not distributive (refer to the lecture notes on Boolean algebra for definitions).

(d) Fill in the proof of Boole's Expansion theorem (page 19 of the Boolean algebra lecture), i.e. Given a Boolean algebra $(\mathbf{B}, +, \cdot, 0, 1)$, show that if $f : \mathbf{B}^n \to \mathbf{B}$ is a Boolean function, then

$$f(x_1, x_2, \ldots, x_n) = \\ x_1' \cdot f(0, x_2, \ldots, x_n) + x_1 \cdot f(1, x_2, \ldots, x_n)$$

for $\forall (x_1, \ldots, x_n) \in \mathbf{B}^n$.