EE219B: Homework 1

Professor Robert K. Brayton

Due January 30 (Tuesday) in class

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1. We say a variable x is in the **support** of a Boolean[1] function f provided $f_x \neq f_{\overline{x}}$. How many Boolean functions of three variables have three variables in their support? Note that a Boolean formula involving three variables does not necessarily imply that all three variables are in the support of the function denoted by the formula.

2. Prove the following properties of cofactor.

   (a) $(f+g)_x = f_x + g_x$

   (b) $(fg)_x = f_x g_x$

   (c) $(\overline{f})_x = \overline{(f_x)}$

3. If F (a set of cubes $\{c_1, \ldots, c_k\}$) is a cover for f, prove that F_x is a cover for f_x, where x is any literal.

4. Prove that every prime of a unate Boolean function is essential.

5. The function $f = x_1x_2x_3 + x_4x_5x_6 + \ldots + x_{3n-2}x_{3n-1}x_{3n}$, has $3n$ variables in its support and is called the *Achilles Heel* function. How many cubes does the minimum irredundant SOP expression for $\overline{f}$ have?

6. Consider the function $f(x_1, x_2, \ldots, x_n) = x_1 \oplus x_2 \oplus \ldots \oplus x_n$, where $\oplus$ denotes XOR (exclusive OR). Convince yourself that f is the odd-parity function on n variables. Is the function unate? How many minterms does it have? How many primes does it have? How many essential primes?

7. Let f be a function $f : \{0,1\}^n \rightarrow \{0,1\}$. Prove that the number of essential primes of f is $\leq (2^{n-1})$. (To get the right idea consider the case $n = 3$.) Give an example of a function that has 2^{n-1} essential primes

*Office hour: M5-6 in 550-D. Call 2-3234 from the phone outside 550, if you don't have access to 550 Cory.

[1]In this course Boolean means $\{0,1\}$

8. Use Tautology checking and the Unate Recursive paradigm to check if the cover F contains the cube C, i.e. $F \supseteq C$.

$$C = ade' \quad (1)$$

$$F = \{b'c'd, bce', ab'c, acd', a'be, abc'd, c'de\} \quad (2)$$